STUDY OF THE PROPAGATION OF γ-RADIATION IN MATTER BY THE METHODS OF THE THEORY OF SIMILARITY

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The main principles of the application of the theory of similarity to problems of the passage of nuclear radiation through matter (e.g., shielding) are discussed. The similarity criteria are obtained in general form and in relation to specific examples, and it is shown that, even in cases where exact similarity does not obtain, the method gives reasonable predictions for a wide range of variation of the parameters.

The design of equipment employing nuclear radiation sources involves the solution of various problems on the passage of radiation through matter. For purposes of generalizing the theoretical and experimental data and providing a genuine scientific basis for modeling, it is useful to apply the theory of similarity, which is widely used in various fields of science and technology [1-3].

Physical similarity is inherent in the problem of the passage of radiation through matter and, in fact, has been used by a number of authors [4-7], but without reference to the theory of similarity itself, which was first applied by Guberman [8] to problems of nuclear geophysics. However, Guberman's examples relate to a point source of radiation. The present paper is concerned with extended sources, more attention being given to the effect of boundary conditions and the physical content of the similarity criteria in a series of cases. Approximate similarity is also considered in greater detail, and for cases when the conditions of approximate similarity are not fulfilled, the concept of quasi-similarity is proposed.

The theory of similarity defines the form of the generalized variables (criteria) in which the problem must be considered, while the relations between these variables are determined from experiment or calculation. Let us consider the kinetic equation of the propagation of γ-quanta in matter

$$\frac{\partial N(r,\ \Omega,\ E,\ t)}{\partial t} + \operatorname{div}[\Omega N(r,\ \Omega,\ E,\ t)] + \mu(E) N(r,\ \Omega,\ E,\ t) = \\ = \iint N(r,\ \Omega,\ E,\ t)\, n\, \sigma(\Omega' \to \Omega,\ E' \to E)\, dE'\, d\Omega' + S(r,\ \Omega,\ E). \tag{1}$$

From (1) we obtain the similarity criteria μr, μ_k/μ, Sr/N, E/E_0, ct/r. If Φ_i is an unknown quantity, then in general form

$$\Phi_i = \Phi(\mu r,\ \mu_K/\mu,\ Sr/N,\ E/E_0,\ ct/r), \tag{2}$$

where the subscript i denotes the investigated points and the physical characteristics of the investigated media. We note that Φ_i may stand for any of the quantities characterizing a specific problem, for example, $N(r_i, E)$, $I(r_i, E)$, $D(r_i)$, $D_a(r_i)$, $D_aV(r_i)$. When it is impossible to select natural geometric and time scales, a scale parameter should be sought from the physical conditions of the problem. For instance, in the case of a point source of γ-quanta in an infinite medium, the only possible extension scale is the reciprocal of the absorption coefficient. Thus, the quantity μr defines like points of similar systems, and, if generalized quantities are used, all the geometric characteristics must be expressed in units of μr. When nonstationary problems are considered, a natural time scale is also lacking. In this case, the quantity $1/\mu c$ should be used as a scale, i.e., the criterion μct determines the comparability in time of points of similar systems. We shall, however, consider stationary systems only.

A necessary feature of similar systems is geometric and physical similarity, and also similarity of boundary conditions. Geometric similarity implies that radiation sources must be geometrically similar (shape of source and angular distribution of radiation) and that similarity in the distribution of sources in the system must be maintained. The shape of the complete system and also of elements of the system with different physical properties (inhomogeneity) must be described by identical equations. The criterion of physical similarity is μ_k/μ, which is identical at all like points, i.e.,

$$\mu_K/\mu = \text{idem}. \tag{3}$$

Physically, this ratio is a measure of the contribution of Compton scattering to the total attenuation of γ-radiation in the medium (if we consider combinations of energies and media for which only photoabsorption and Compton effect are important).

Thus, when geometric similarity is present and the condition μ_k/μ = idem is satisfied, systems are similar. Then, for the required criterion we may also write Sr/N = idem. This criterion is written differently depending on the conditions of the problem. If the field outside a radiation source is being studied, then the criterion takes the form S/Nr^2. If the source is not a point source, then for it, too, all the conditions of similarity (geometry, radiation characteristics, absorption properties of source material) must be satisfied. If the field inside a radiating medium is studied, then the form of the criterion remains as before, i.e., Sr/N. In the case of uniformly distributed sources in an infinite homogeneous medium, the idea of like points defined by values of the criterion μr loses significance. In the theory of similarity, this case, when similarity is preserved at all points of the system irrespective of the corresponding criteria, is called self-similarity. However, if the medium is finite, then there is no self-similarity close to the boundaries, and the geometry of the systems and the external conditions must be similar.

It is easy to show that requirement (13) is satisfied if the systems have the same effective atomic number. If, however, in addition, $(Z/A)' = (Z/A)''$, then $r'/r'' = \mu''/\mu' = \rho''/\rho' = \alpha$. Hence, the coordinates of like points in two similar systems are related by $r' = \alpha r''$, where $\alpha = \rho''/\rho'$. Then, from S/Nr^2 = idem, μr = idem, and μ_k/μ = idem, we get an expression relating the fields in two similar systems

$$N_2 = \frac{S_2}{S_1}\frac{N_1}{\alpha^2}, \tag{4}$$

where $\alpha = \rho''/\rho'$ when the atomic numbers of the media are equal, or $\alpha = \mu''/\mu'$ if μ_k/μ is identical for both media over the entire energy interval from E_0 to 0. Under the above conditions the proposed equation allows certain cases of the passage of γ-radiation to be modeled.

Modeling Radiation Sources and Irradiated Objects with Different Geometries

As an example, the table presents the conditions of similarity for systems with different geometries and media. The point source has been studied in detail. Other cases (rod, hollow cylinder) have been investigated less thoroughly, since the rules for point sources may be extended to other configurations with account for their special features. Extended sources are considered as surfaces. Allowance for self-absorption and self-scattering in the sources requires special consideration.

Conditions of Similarity for Systems with Differing Geometries and Media
(K – region of finite dimensions, ∞ – infinite region, $\rho' \gg \rho_0 \ll \rho$)

Source geometry	Extent and density of medium in regions			Extent of radiation sources	Detector located in region.	Similarity parameters
	A	B	C			
Point	∞, ρ	—	—	—	A	r
	K, ρ	∞, ρ_0	—	—	A	r, R
	K, ρ_0	∞, ρ	—	—	B	t, R
	K, ρ_0	K, ρ	∞, ρ_0	—	B	t, R, d
	K, ρ	∞, ρ'	—	—	A	r, R, ρ'
	K, ρ	K, ρ'	∞, ρ_0	—	B	t, R, d, ρ'
Linear	∞, ρ	—	—	∞	A	n
	∞, ρ	—	—	l	A	l, m, n
	K, ρ	∞, ρ_0	—	l	A	l, r, h, m, n
	K, ρ_0	K, ρ	∞, ρ_0	l	B	l, f, t, r, h, R, H
Cylindrical	K, ρ	∞, ρ_0	—	h, r	A	r, h, m, n
	K, ρ_0	∞, ρ	—	h, r	B	r, h, f, t
	K, ρ	∞, ρ'	—	h, r	A	r, h, m, n, ρ'

Approximate similarity. Absolute observation of the fundamental condition of similarity (3) is not always obligatory. If, starting at some energy E_i, this condition is not fulfilled, then, strictly speaking, the laws of similarity will only hold for quantities $\Phi(E)$ where $E > E_i$. However, the error will not always be very substantial. For example, if the detector is separated from the medium by a metallic layer, then it is evident that soft scattered radiation from the medium will be absorbed in the layer, and the detector will register only the hard component of the spectrum with a lower limit $E_{min} = E(r, Z)_{met}$. This barrier must be relatively thin, so that no soft radiation with $E \geq E_{min}$ is generated in it. In this case, only the harder spectral lines are converted, and if the composition of the irradiated medium is such that photoabsorption at these energies is negligibly small, then the requirement that the atomic numbers of the similar media be equal becomes unimportant. Thus, in place of $\alpha = \rho''/\rho'$ for exact similarity ($Z' = Z''$) we get $\alpha = n''/n'$ for approximate similarity ($Z' \neq Z''$). In practice, such a situation may arise in modeling light shielding in a metal chamber, in using isotopic γ-radiation to measure density, moisture content (detector housed in a metal case), in modeling certain irradiation equipment, etc. In these circumstances, the presence in the model of a thin barrier, absorbing the soft part of the spectrum, automatically ensures that the conditions of similarity are observed.

In a number of cases, the need arises to extrapolate experimental or theoretical data to media with Z different from that of the model. Figure 1 shows the attenuation of Co^{60} γ-radiation in a concrete wall obtained by calculations

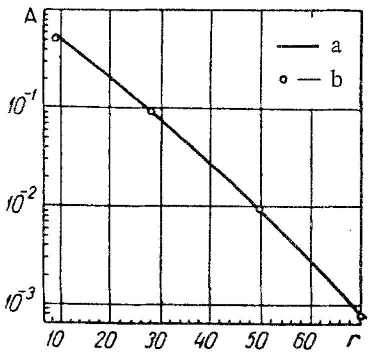

Fig. 1. Attenuation of Co^{60} γ-radiation (A) in a concrete wall (r – wall thickness): a) obtained by similarity from data for aluminum; b) experimental data [10].

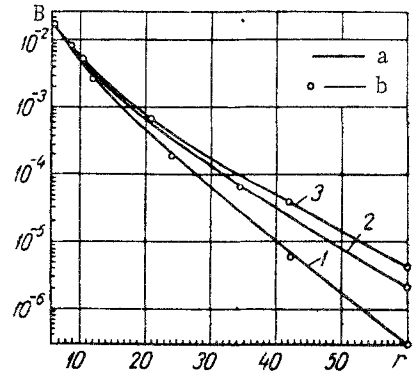

Fig. 2. Dose (B) delivered by a point source of γ-radiation with energy 1, 2, and 3 MeV (curves 1, 2, and 3) behind an aluminum barrier: a) values obtained by similarity from water; b) values obtained by the method of moments.

based on (3) from data for aluminum using a build-up factor [5] and corrections for a finite medium [9], together with experimental data [10]. The values of the criterion μ_k/μ agree for both media down to E = 0.02 MeV; for E = 0.01 MeV this criterion is 0.0077 for concrete and 0.0070 for aluminum. In this connection, the discrepancy between experimental and calculated values of the attenuation does not exceed 3-4%. Violation of the condition μ_k/μ = idem over a wider energy interval may lead to substantial errors. An increase in initial energy is accompanied by an increase in the energy above which the specified condition must be observed, since the relative contribution of the low-energy spectral component, which acts as the source of error, is reduced. At the same time, at sufficiently high initial energies, the laws of similarity may again become inapplicable due to a considerable increase in the pair production cross section and, hence, failure to fulfill the fundamental condition of similarity in the region of energies close to the initial value.

In order to estimate the possible errors involved in using the method of similarity to model radiation equipment, we studied the conversion of the dose fields from a point source for water and aluminum. Data on μ_k/μ for water and aluminum do not coincide over a wide range of energies. However, for initial energies from 2-6 MeV, the discrepancy between the dose obtained for aluminum by similarity from the data for water, and the values obtained by the method of moments does not exceed 4% for $\mu r \leq 10$. For E_0 = 1 MeV, the discrepancy does not exceed 10% for $\mu r \leq 3$, and 20-25% for $\mu r \leq 10$. The results for initial energies of 1, 2, and 3 MeV calculated from (4) are shown in Fig. 2.

In the irradiation of chemical and biological objects, the densities involved do not usually exceed 2 g/cm^3. In the energy range of interest ($E \geq 0.05$ MeV), the values of μ_k/μ for the commonest irradiated materials are quite similar. Hence it is obviously useful to apply the theory of similarity to the study of the propagation of γ-radiation through these materials. Clearly, the model medium for the majority of chemical irradiations will be water or, in some cases, aluminum. To estimate the possible error of conversion, we made a Monte Carlo calculation of the absorption of γ-radiation from a Co^{60} source located at the center of a hemisphere of radius μr = 3. Water and aluminum were chosen as the modeling media. It was found that the fraction of absorbed dose due to photoabsorption did not exceed 10-12%. In the case of iron, the corresponding value was found to be within 20%. Calculation of the distribution of dose absorption in the hemisphere showed that the error associated with conversion from water to aluminum does not exceed 5-7%.

Quasi-similarity. In shielding design, frequent use is made of materials composed of elements whose atomic numbers differ considerably. The use of data obtained for these materials by the methods of exact or approximate similarity may lead to serious errors, since the values of μ_k/μ are clearly different even at quite high energies. It is natural to assume that in this case the likely error is a function of the values of μ_k/μ for these media. To allow for this error, we introduce into (2) and (4) the correction factor

$$\xi = \xi \left[\left(\frac{\mu_\kappa}{\mu} \right)_{Z_i,\, E_i},\ E,\ E_0,\ \mu r \right]. \tag{5}$$

Then

$$\Phi_2 = \frac{S_2}{S_1} \frac{\Phi_1}{\alpha^2} \xi. \tag{6}$$

The value of the factor ξ is determined by the degree of departure from the conditions of similarity. We have called this modified similarity "quasi-similarity." The form of the argument $(\mu_k/\mu)_{Z_i, E_i}$ in (5) should be more clearly defined. First, the noncorrespondence between the μ_k/μ for different media should be taken into account over the entire energy interval studied. Then, the argument $(\mu_k/\mu)_{Z_i, E_i}$ may be replaced by two new arguments

$$\Delta_E = \left(\frac{\mu_K}{\mu}\right)'_E \Big/ \left(\frac{\mu_K}{\mu}\right)''_E \text{ and } C_E^{E_0} = \left(\int_E^{E_0} \left(\frac{\mu_K}{\mu}\right) dE\right)' \Big/ \left(\int_E^{E_0} \frac{\mu_K}{\mu}\, dE\right)'',$$

where the single and double primes relate to the two different media. Hence

$$\xi = \xi(\Delta_E,\ C_E^{E_0},\ E,\ E_0,\ \mu r). \quad (7)$$

The form of (7) depends on what is understood by Φ_i in each concrete case $- I(r_i)$, $D_a(r_i)$, etc. Here, as always, the theory of similarity supplies only the set of arguments, on which the unknown function depends. The functional dependence itself must be found from experimental or numerical data. As an illustration, certain cases in which data on the energy spectrum are treated by the proposed method are presented below.

If the spectral composition of γ-radiation from a point isotropic source in an infinite aqueous medium $I(r, E)_{H_2O}$ is known, then corresponding data for other media may be obtained from (6) using the formula:

$$I(r,\ E) = \xi I(r,\ E)_{H_2O} (\mu/\mu_{H_2O})^2. \quad (8)$$

Analysis of the data of [5] showed that

$$\xi = \left(\frac{\Delta_E}{C_E^{E_0}}\right)^2 (\mu r)^{\frac{(1.66 \lg E_0 - 1) A(E) + 0.07}{62} \rho^2}. \quad (9)$$

Here
$$A(E) = 0.48 - 1.60E \text{ for } 0.1 < E < 0.22 \text{ MeV},$$
$$A(E) = 0.15 - 0.11E \text{ for } 0.22 < E < 0.60 \text{ MeV},$$

If $E \geq 0.6$ MeV, then $\xi = 1.0$. Equation (8) is valid for materials whose atomic numbers do not exceed the atomic number of iron, $E = 0.5$-20 MeV, $\mu r = 1.0$-20.0. Calculations showed that in most cases the error in using (8) does not exceed 10%, or 20% for $\mu r \geq 10$. The validity of the equation for $\mu r > 20$ was not checked.

Other data relating to the propagation of γ-radiation from sources of different shapes and sizes, with arbitrary angular distribution of the emitted radiation, etc., may be treated analogously. The limits of applicability of the expressions obtained are established in each individual case. Extended radiation sources may also be considered.

Other Types of Radiation

The methods of the theory of similarity may be used for different types of radiation. Apart from "pure" radiation sources (γ-quanta, neutrons, electrons, etc.), the theory may be used to study, for example, the distribution of capture γ-quanta in neutron irradiation. As an example of the application of the theory to the study of neutron distribution, we have calculated the spatial distribution of fast neutrons in polyethylene. Plexiglas was chosen as the modeling medium. In this case, α is written in the form $\alpha = (\sigma_1/\sigma_2) \cdot (\rho_1/\rho_2)$. However, for neutrons there are additional constraints related to the different form of the kernel of the integral operator of the kinetic equation for different materials. The results of calculations using experimental data are shown in Fig. 3; the discrepancy between calculation and the experimental results does not exceed a few percent.

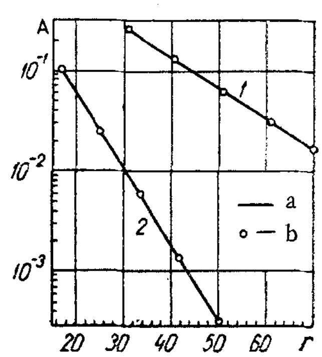

Fig. 3. Spatial distribution of fast neutrons in polyethylene $A \equiv 4 r^2 N$ obtained by conversion from data for plexiglas. 1) 1.49 MeV; 2) 4 MeV; a) values obtained by similarity; b) experimental values.

NOTATION

$N(r, \Omega, E, t)$ — number of γ-quanta with energy in the interval E to $E + dE$ passing in unit time through unit area at a point r through an element of solid angle $d\Omega$ about a direction Ω; $\mu(E)$ — linear γ-ray absorption coefficient; n — number of electrons per unit volume; $\sigma(\Omega' \to \Omega, E' \to E)$ — differential cross section for the transition of quanta from the state (Ω', E') to the state (Ω, E); $S(r, \Omega, E)$ — number of quanta emitted by source in unit time; μ_k — Compton scattering coefficient; E_0 — initial energy of γ-quanta; c — velocity of propagation of γ-quanta in medium; $N(r_i, E)$ — flux of γ-quanta; $I(r_i, E)$ — γ-ray energy flux; $D(r_i)$ — radiation dose; $D_a(r_i)\, D_{aV}(r_i)$ — dose absorbed in irradiated volume; r_i — set of geometric parameters of system; r — characteristic dimension; Z — atomic number of medium; A — atomic weight of medium; ρ — density of medium; superscripts ' and " refer to two similar systems.

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